Capability Satisficing in High Frequency Trading

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Abstract:

HFT firms are neither exactly expected utility maximizers nor profit maximizers. They are averse to both price risk and demand risk. This paper explains the capability theory of how these firms make allocation decisions under uncertainty, and shows how capability maximization is precisely consistent with utility theory. The issue, however, is how these firms actually make allocation decisions in practice. Using the Gioia methodology, this paper presents evidence from interviews with HFT professionals and specialist media that suggests that these firms are capability satisficers. Capability theory is also consistent with bounded rationality and the adaptive markets hypothesis, and defines the point at which these firms reach a satisfactory solution. Thus, capability reconciles mainstream theory and the more realistic, behavioral theories based on observation of industry practice. The methodology developed can be applied to any firm that makes algorithmic decisions under uncertainty.

Keywords: High frequency trading, adaptive markets hypothesis, capability theory, satisficing

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1. INTRODUCTION

> *Finance is the application of economic principles to decision-making that involves the allocation of money under conditions of uncertainty. Investors allocate their funds among financial assets in order to accomplish their objectives.* – Fabozzi and Drake [2009]

Mean-variance expected utility has been the foundation of decision theory in mainstream finance for decades. But, the advent of automated trading has changed everything and new theories are needed [see O'Hara 2015]. Today, the markets are ecologies of algorithms [Hasanhodzic et al. 2011; Farmer and Skouras 2013; MacKenzie 2014]. These algorithms encapsulate trading strategies that, presumably, have some efficacy. Among these, high frequency strategies alone account for over 70% of the daily trading volume [Brogaard 2010; Hoffman 2014].

The mainstream theories have a difficult time with high frequency trading (HFT). The profits some of these strategies earn and the consistency with which they earn them are inconsistent with the efficient markets hypothesis. This leads some to conclude either that those theories are wrong or that HFT firms (henceforth firms) are doing something nefarious. Behavioral theories in finance also have a difficult time with HFT. The systems these firms create do what Richard Thaler says people cannot do—"calculate like a computer and have no self-control problems[1] [see Orrel 2010]." While this may be true, Kumiega and Van Vliet [2012] argue that behavioral biases in the management of algorithmic strategy research and development projects and on/off allocation decisions open the door to (potential) irrationality.

Whatever the case, this paper overcomes these difficulties by starting from first principles and arguing these firms are something new. They are neither pure, market-risk-averse utility maximizers (as in mainstream finance theory) nor pure, risk-neutral profit maximizers (as in theory of the firm). They are a combination of both, because they are sensitive to both market risk and demand risk. They are in theory something called capability maximizers (from the perspective of mainstream finance and a rational, idealized world) or they are capability satisficers (from the perspective of behavioral finance and a real, boundedly rational world) in the sense of Simon [1965] (henceforth Simon). Thus, the concept of capability allows us to "move back and forth from the world of theory to the world of action [Ostrom 1990]," by reconciling the mainstream-finance / behavioral-finance dichotomy. It does so by taking the evolutionary, or biological, approach to markets [see Farmer and Lo 1999; Farmer 2002; Farmer and Skouras 2013] under the adaptive markets hypothesis (AMH) of Lo [2004; 2005], where "financial agents compete and adapt, but not…in an optimal fashion [Lo 2007]."

Empirically, while HFT firms are notoriously secretive, I was able to follow the methodology of Gioia et al. [2012] and interview seven HFT industry professionals. These interviews shed light on emerging issues on how these firms compete and adapt. The aim is to contribute to the theoretical and case study literatures on HFT by taking a cross-disciplinary view. While the sample is small, common concepts and themes across firms appear to support the aggregate dimension that (rather than optimizing) these firms are capability satisficers. They allocate to strategies that achieve some target, goal, or aspiration level, or simply that are "good enough." Quotes drawn from the specialist media corroborate these interviewees' comments.

Relative to the existing literature, this paper makes four contributions. One, this paper presents capability as a descriptive theory of decision-making in HFT under the AMH. Two, this paper extends an existing profit function for HFTs and re-presents capability using the more intuitive statistics of the distribution of the sample sum, rather than the sample mean. Third, this paper extends capability theory by explaining these firms' time *and* risk preferences according to a ratio of measurable components—the Cooper ratio—that shows that capability is precisely consistent with utility maximization. Four, as maximization is not possible in practice, this paper presents qualitative evidence that these firms exhibit satisficing behavior when making on/off allocation decisions.

The remainder of this paper proceeds as follows. Section 2 provides some background concepts, including a review of the relevant literature, the economic premise of capability theory, and an update to the HFT profit model. Section 3 presents a numerical implementation of an example HFT strategy. Section 4 redefines capability using the distribution of the sample sum. Section 5 connects capability to utility maximization. Section 6 presents the evidence for the proposition that these firms are in practice capability satisficers. Section 7 concludes.

2. BACKGROUND AND LITERATURE REVIEW

In mainstream finance theory, Von Neumann and Morgenstern's [1947] expected utility theorem describes people's preferences with regard to decision making under uncertainty under the assumption of rationality. Further, it accounts for risk aversion, meaning people's utility functions are concave. This leads to expected utility maximization and the measure of relative risk aversion λ of Arrow [1965] and Pratt [1964]. The first order condition of optimality of expected utility leads to the well-known Sharpe ratio (*SR*) [Sharpe 1966], which is commonly used to rank the performance of portfolio strategies, where $SR = (E(r) - r_f) / \sigma$. For *any* investor, the higher the *SR*, the higher the expected utility, regardless of their objectives or their idiosyncratic level of risk aversion λ.

Kumiega et al. [2014] suggest that the performance of HFT strategies is more appropriately assessed against a capability ratio C_{pl} as in equation (1), which measures of the ability of a process to satisfy a lower specification limit [Kane 1986]. In (1) μ_n is the average expected return per trade, σ_n is the standard deviation of expected returns per trade, and c is the specification limit, which is the average fixed cost necessary to build and operate these infrastructure-intensive strategies.[2]

$$C_{pl}(n) = \frac{\mu_n - c}{3\sigma_n} \geq 1 \tag{1}$$

Cooper et al. [2015] adds that non-normality in the returns of strategies can be more precisely modeled and that capability can be a proxy for prudence in algorithmic trading.

These authors present capability as a prescriptive framework, combining statistical control and risk-adjusted performance measurement. Firms *ought* to invest in algorithmic strategies with capability ratios that are greater than one. Their assumption is that stationary *trading profits* are available to firms that invest in the correct combination of technologies and algorithms. Consistent profitability is a stylized fact of HFT [see for example, Baron et al. 2012; Kirilenko et al. 2015], and it is what justifies these firm's large, up-front investments in research and technological infrastructure.

In behavioral economics, Simon's contributions on bounded rationality and satisficing form the foundation of theories derived from observation of agents [Schwartz 2002]. A wide body of literature develops these concepts, including some contributions related to the market ecology of algorithms. Brennan and Lo [2012] develop a binary choice model of decisions and show that "bounds on rationality are determined by physiological and environmental constraints." Hasanhodzic et al. [2011] "suggest that a reinterpretation of market efficiency in computational terms might be the key to reconciling [the efficient markets hypothesis] with the possibility of making profits based on past prices alone…It does not make sense to talk about market efficiency without taking into account that market participants have bounded resources."

In addition to the literature on the AMH already discussed, other authors in finance have also addressed satisficing, the "decision-making procedure or cognitive heuristic that entails searching through the available options just long enough to find one that reaches a preset threshold of acceptability [Colman 2015]." Firms satisfice in ambiguous (i.e. complex and dynamic) environments that preclude any sort of optimization or optimal behavior. Lanzillotti [1958] and Payne et al. [1980], and Payne et al. [1981] show that managers are concerned with profit targets, not profit maximization. Abbas et al. [2008] and Bordley and Kirkwood [2004] investigate "target-oriented utility." The literature of engineering describes a similar concept, "goal-oriented requirements engineering," where "a goal is an objective the system under consideration should achieve" [see Van Lamsweerd 2001]. However, none of these address satisficing in the highly complex and dynamic environment of HFT.

Most importantly for this paper, Brown and Sim [2009] develop the idea of a satisficing measure, "which depends on a [risky] position's performance relative to a given aspiration level." These authors extend their ideas in Brown et al. [2012] to "provide some formalism for the role of aspiration levels in choice under uncertainty." These articles assume diversification among positions, or "acts," and convex preferences. Under their definition, the *SR* is a satisficing measure on a linear space of a finite set of random variables (even though it does not define an aspiration level), because it satisfies the axioms of attainment content, nonattainment apathy, monotonicity, and gain continuity. I apply their definition to the HFT case to show that the capability ratio is also a satisficing measure.

2.1 THE ECONOMIC PREMISE OF CAPABILITY

The economic premise of capability theory follows the evolutionary and computational approach of Hasanhodzic et al. [2011]. As such, it assumes that HFTs are suppliers of efficient markets (with respect to short-term[3] information) and liquidity to longer-term traders and investors who demand these characteristics of markets. Whether this view holds or not is the subject of debate. However, the preponderance of empirical evidence supports the hypothesis that HFT aids in price discovery and increases liquidity [Brogaard 2010; Brogaard et al. 2014; Hasbrouck and Saar 2013; Jovanovic and Menkveld 2011; Riordan and Storkenmaier 2012]. More specifically, I assume:

1. HFT strategies generate revenues, or gross profits, by extracting private information from public market data and incorporating it into future price changes of financial instruments [see for example, Grossman and Stiglitz 1980; and Benos et al. 2015]. This revenue represents the price that longer-term traders and investors are willing to pay for removal of a short-term inefficiency, or opportunity O [see Zhang 1999].
2. HFT strategies also generate revenues, or gross profits, through the use of passive limit orders, thereby supplying liquidity and earning the bid-ask spread S. The spread consists of a fee for liquidity provision plus a premium for assuming the risk of adverse selection when transacting against informed traders [Glosten 1987].

The AMH argues that evolutionary processes at work in the markets will determine the correct diversity (i.e. number and variety) of HFT strategies that make for a robust ecology.

2.2 THE HFT PROFIT MODEL

Cooper and Van Vliet [2015] attribute trading profits in HFT to the two sources of revenue O and S plus exchange rebates[4] R to build a total revenue TR equation. Because HFTs earn O probabilistically, C is the percentage of the opportunity O that the strategy can capture. Then, revenue from information processing is $C \cdot O$, the captured opportunity, and the total revenue equation is $TR = C \cdot O + S + R$. As I use the term, *trading profit* π is TR minus variable trading costs VC, so that $\pi = TR - VC$. Variable costs are commissions and/or exchange fees and margins, particularly in the case of listed derivatives markets. In equity markets, these firms often pay a flat fee for unlimited trading and may deposit fixed margins. In such a case, these costs are part of fixed costs FC. (Later, I will use the term *net profits* π_{net}, where $\pi_{net} = TR - VC - FC$, and fixed costs are denoted by the letter c.)

In full, then, the expected trading profit *per share* (*or contract*) *traded* $E(\pi)$ of an HFT strategy is defined in equation (2).

$$\mu = E(\pi) = O \cdot C + S + R - VC \quad (2)$$

The expected trading profit *per trade* is the trading profit per share traded times the number of shares traded, or the size s, as $E(\pi_t) = \mu_t = \mu \cdot s$. Given a quantity of trades q per day (a proxy for demand for the information processing and liquidity they supply through passive limit orders), then the expected trading profit[5] *per day* $E(\pi_d) = \mu_d = \mu_t \cdot q$. Given many trading days, or a waiting time w, the expected trading profit *per w* is $E(\pi_w) = \mu_w = \mu_d \cdot w$. Of course, μ, μ_t, μ_d, and μ_w are simply expectations of four distributions of trading profit outcomes, and q is itself a random variable (as is potentially s). The shape of the distribution of profits per share traded around the expectation in (2) determines the shapes of the others through the central limit theorem. These firms take advantage of these relationships to increase profits and decrease risk of loss.

Appendix A provides a brief review of the central limit theorem and the statistical relationships between a distribution and the distributions of its sample mean and sample sum. In HFT, the distribution of the sample sum of trading profits is the powerful concept. Because the mean grows faster than the standard deviation, *any* trading strategy that has a positive expectation will be increasingly profitable and with an increasing level of reliability of profitability as q and w increase. This is related to the concept of "success probability" in Browne [1999], Follmer and Leukert [1999], and Muller and Baumann [2006]. The best way to demonstrate this is through a numerical implementation.

3. NUMERICAL IMPLEMENTATION

Consider a (trivial) equity HFT strategy that implements a short-term directional forecast or intermarket arbitrage or statistical arbitrage using passive limit orders (i.e. make-make tactics [see Cooper and Van Vliet 2015]) and holds no positions over night. Such a strategy may in reality have a (virtually) stationary and non-normal reference distribution of trading profits per share traded π, with a small expectation (say, just one tenth of a penny, \$0.001) and a long left tail in cases of adverse selection, so that $\mu = 0.001$, $\sigma^2 = 0.0004$, $\gamma = -3.0$, and $\kappa = 33.0$. Figure 1 shows this reference distribution of trading profits per share traded.

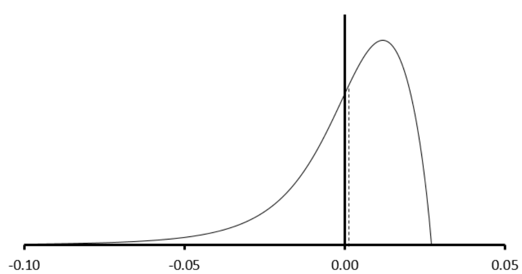

Figure 1: Reference Distribution of Trading Profits per Share Traded

If the strategy trades $s = 100$ shares per trade, then the distribution of trading profits per trade has the moments $\mu_t = 0.10$, $\sigma_t^2 = 4.00$, $\gamma_t = -3.0$, and $\kappa_t = 33.0$. Incidentally, these moments show why these firms prefer to make many small trades as opposed to fewer larger ones, which liquidity demanders prefer [Easley and O'Hara 1987]. For a given volume of shares traded, or scale $k = s \cdot q$, the variance over k is $\sigma_k^2 = \sigma^2 \cdot s^2 \cdot q$. Thus, the variance of a single one thousand share trade, $\sigma_k^2 = \sigma^2 \cdot 1000^2 \cdot 1 = \sigma^2 \cdot 1{,}000{,}000$, is ten times greater than the variance of executing ten one hundred share trades, $\sigma_k^2 = \sigma^2 \cdot 100^2 \cdot 10 = \sigma^2 \cdot 100{,}000$. This is consistent with the empirical evidence. As HFT activity has increased, the average trade size has gone down [see Kumiega et al. 2016].

If the strategy makes many of these $s = 100$ trades per day, say $q = 10$, 100, or 1000, respectively, then the three resulting distributions of profits *per day* are shown Figure 2. The distribution of the sample sum is very close to normal, especially at $q = 1000$, the right-most graph. Given this distribution, which has the moments $\mu_d = 100.00$, $\sigma_d^2 = 4000.00$, $\gamma_d = -0.095$, and $\kappa_d = 0.033$, the firm has a high probability of generating between \$0 and \$200 in trading profits per day from this strategy.

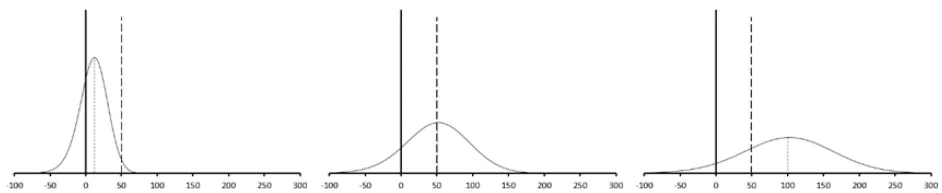

Figure 2: Distributions of Trading Profits per Day
$s = 100$, $q = 100$, 500, and 1000

If the strategy is making (an average of) $q = 1000$ trades per day and runs for several days, say $w = 5$, 10, or 20 days, then the three distributions of profits per waiting time w are shown in Figure 3. The distribution of the sample sum where $w = 20$ has the moments $\mu_w = 2000.00$, $\sigma_w^2 = 80000.00$, $\gamma_w = -0.021$, and $\kappa_w = 0.002$. Given this distribution, the firm has high probability of generating trading profits of between \$1500 and \$2500 per month from the strategy.

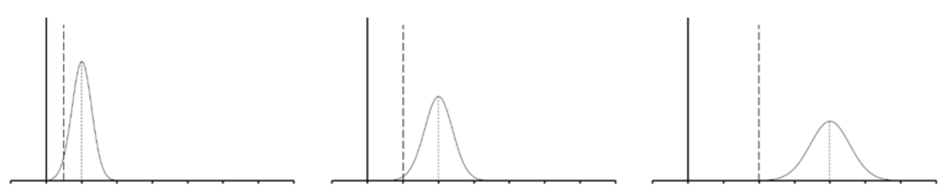

Figure 3: Distributions of Trading Profits per Period
$s = 100$, $q = 1000$, $w = 5$, 10, and 20 days

The heavy, vertical, dashed lines in each chart in Figure 2 and Figure 3 represent the fixed cost $c = FC$ associated with the trading strategy (plus any required rate of return) that the firm hopes to exceed by allocating to the strategy. In Figure 2, this cost is $50 per day in the three charts. In Figure 3, this cost is $50 × 5 = $250 per week, $50 × 10 = $500 per two-week period, and $50 × 20 = $1000 per month.[6] Because the fixed costs in HFT are so large and cannot be assumed away, expected *net profits* π_{net} are what really matter, where $\pi_{net} = \mu_w - c_w$. As a goal formulation, these firms search for "capable" strategies, as defined in Proposition 1, which is self-evident.

Proposition 1: The objective, or goal, of the HFT firm is to allocate capital to capable strategies; ones it believes will be able to reliably generate net profits over some acceptable timeframe.

The risk in Proposition 1 and that these firms are averse to is the probability of loss over a given period w, as in Mao [1970]. Lower reliability of profits means greater probability of loss and consecutive losing periods, which raises doubt about the moments of the distribution and may lead to bankruptcy [see Cooper et al. 2015]. This arises as much from uncertainty about demand, or quantity q, as it does from price uncertainty σ.

This is why low latency technology is so important. "The fastest firms (in absolute and in relative terms) earn the highest profits [Baron et al. 2012]." Technological speed is necessary to increase C in order to achieve positive μ, minimize the length of the left tail (i.e. lower adverse selection risk), *and* to increase q to a level that makes the strategy reliably profitable. Faster technological speed means better position in the limit order queue, and the better the position in the limit order queue, the more often the strategy can earn O (because O is fleeting) and S, and the relatively less often it realizes adverse selection.

4. DEFINING CAPABILITY

Capability theory models Proposition 1 by conveying the ability of a strategy to generate net profits π_{net} over the firms' acceptable w with sufficient reliability. This is to say that, unlike equation (1), these firms have idiosyncratic preferences when it comes to time *and* risk of loss. Their time preference w must be something less than 252 days. They are obligated to pay rent, salaries, and allocate IT costs out of trading profits at regular intervals, say biweekly or monthly. Over w, the probability of loss[7] is $P(\pi_w < c_w)$, or put the other way, the reliability of net profits, or the success probability, is $1 - P(\pi_w < c_w)$. Implicitly or explicitly, the firm has some tolerance ξ for loss over w, or a required reliability of net profits $1 - \xi$. The definition of capability in (1) prescribes that firms use $\xi = 0.00135$ (i.e. $z = 3$), but this is not necessarily the case for all firms. For example, relative to the frequency distribution of trading profits μ_w, a tolerance for loss of $\xi = .05$ would equate to a z-score of $\Phi^{-1}(95\%) = 1.64$ times the standard deviation of trading profits σ_w.

The intuition of capability is most easily grasped visually. Figure 4 depicts the distribution of μ_w using the right most chart from Figure 3 where $q = 1000$ and $w = 20$. As can be seen, the length of the line π_{net} is the expected net profit. The line $z\sigma_w$ is the distance from the mean to the level that corresponds to the firm's ξ, its acceptable probability of loss. The remainder $\varphi = \mu_w - z\sigma_w - c_w$ is the safety margin, similar to that described in Fellner [1948] and Day et al. [1971]. So, if the length of π_{net} is greater than the length of $z\sigma_w$, then the strategy is capable. The firm can expect the strategy to generate net profits π_{net} reliably over its acceptable w.[8]

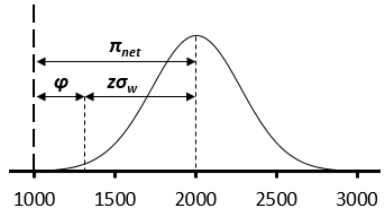

Figure 4: Capability of the HFT Strategy
***s* = 100, *q* = 1000, *w* = 20**

The capability ratio $C_{pl}(\ \xi,\ w\ )$ in (3) conveys this relationship. (For additional discussion, see Appendix B.)

$$C_{pl}(\xi, w) = \frac{\mu_w - c_w}{\Phi^{-1}(\xi)\,\sigma_w} = \frac{\pi_{net}}{z\sigma_w} \geq 1 \tag{3}$$

Again, the objective, or goal, or aspiration, is that a strategy achieve of $C_{pl}(\ \xi,\ w\ )$ level of one. Relative to (1), the specification of C_{pl} in (3) uses the sample sum instead of the sample mean and does not prescribe that all firms' $z = 3$. For the example strategy used earlier and using the moments from the distribution in Figure 4, if the firm's $\xi = 0.05$ and $w = 20$, then the strategy is capable since $C_{pl}(\ .05,\ 20\ ) = (\ 2000 - 1000\ ) / (\ 1.64 \cdot \sqrt{80{,}000}\ ) = 2.16$.

Relative to the Proposition 1, capability is a reasonable representation of the decision-making problem in HFT. However, rather than simply arguing that firms *ought* to allocate capital to capable strategies, what is it they actually do? From which side of the mainstream-finance / behavioral-finance dichotomy can or should we perceive these firms? While a higher capability ratio is always better, do these firms maximize capability (in the Von Neumann-Morgenstern and EMH sense) or simply allocate capital to those strategies that achieve the aspiration level (in Simon's satisficing sense and the AMH).

5. CAPABILITY MAXIMIZATION

Given the variables discussed thus far, Table 1 shows their directions that increase the value of a strategy's capability ratio, and these make intuitive sense. These firms prefer more trading profit, less market risk, smaller trade size, greater quantity, and lower fixed costs, while the firm's time and risk preferences *w* and ξ are constraining.

Trading Profit	π	↑	
Market Risk	σ^2	↓	
Size	s	↓	
Quantity	q	↑	
Fixed Cost	c	↓	
Waiting Time	w	↓	← Constraint
Probability of Loss	ξ	↓	⎫
Reliability of Net Profits	$1 - \xi$	↑	⎬ Constraint
	$\Phi^{-1}(1 - \xi) = z$	↑	⎭

Table 1: Capability Maximizing Directions

For these firms to be rational agents, capability maximization must be in some way a form of or consistent with utility maximization. Indeed, this is the case.

Proposition 2: In mainstream finance theory, HFT firms maximize capability.

Cooper et al. [2015] (henceforth just Cooper) show that the C_{pl} in (1) can be decomposed into a *daily SR* times $\sqrt{w} / 3$ using the distribution of the sample mean. Here, what I call the daily SR^{HFT} employs the distribution of the sample sum given *q*, and daily fixed costs c_d (instead of r_f), so that $SR^{HFT} = (\mu_d - c_d) / \sigma_d$. Thus, the C_{pl} in (3) is the same as the SR^{HFT} multiplied by what I call the daily Cooper ratio $CR = \sqrt{w} / z$ as shown in equation (4)[9].

$$\begin{aligned} C_{pl}(\xi, w) &= \frac{\pi_{net}}{z\sigma_w} && \geq 1 \\ &= \frac{\mu_d - c_d}{\sigma_d} \cdot \frac{w}{z\sqrt{w}} = SR_d^{HFT} \cdot \frac{\sqrt{w}}{z} && \geq 1 = SR^{HFT} \cdot CR \geq 1 \\ &= \frac{\mu_t - c_t}{\sigma_t} \cdot \frac{qw}{z\sqrt{qw}} = SR_t^{HFT} \cdot \frac{\sqrt{qw}}{z} && \geq 1 \end{aligned} \tag{4}$$

The *CR* combines the firm's idiosyncratic time and risk preferences into a single measure. Figure 5 depicts the surface of the *CR* over the various combinations of *z* (from .25 to 3) and *w* (from 5 to 50) given *q* = 1000. As can be seen, high values of *w* and low values of *z* result in high values of *CR*. Multiplying SR^{HFT}s of strategies by a high *CR* toward the back corner results in more strategies being capable (and therefore, warranting allocations), but the impetus of the firm is to push its *CR* as far toward the lower front corner as possible. Thus, there is an implicit trade-off between w and ξ. In the numerical example, the SR^{HFT} = (100 – 50) / √4000 = 0.79, and the *CR* = √20 / 1.64 = 2.73, so that the C_{pl} = 0.79 × 2.73 = 2.16 as before. The black dot in Figure 5 represents this value of the *CR*.

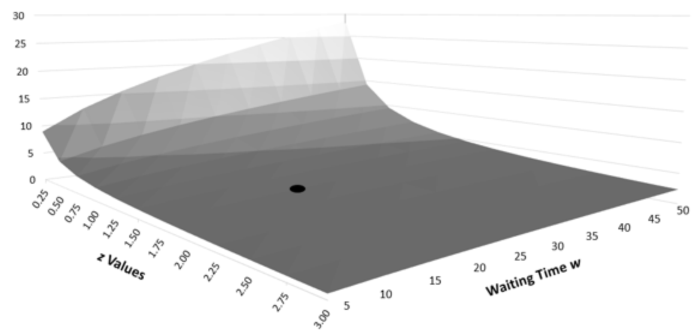

Figure 5: Surface of the Cooper Ratio
***q* = 1000**

Maximizing the SR^{HFT} alone assumes that that all firms' *CR*s equal one for all their strategies. This only happens if all these firms' tolerances for risk of loss are equal to the square root of their time preferences, or $z = \sqrt{w}$ for a given level of q. This is unlikely to be true for all firms, or for all their strategies. The actual *CR* depends upon each firm's willingness to accept losses over some timeframe. Which is to say that, unlike the coefficient of risk aversion λ, it should be possible to measure more directly these firms' preferences, if only these firms were not so secretive.[10] For example, as these firms are often privately held, their investors may very well specify their tolerances for consecutive losing months in advance.

Given the decomposition in (4), we can say that C_{pl} maximization occurs at the maximum SR^{HFT} times the firm's *CR*. Thus, the C_{pl} is an affine transformation of the SR^{HFT} and is therefore "precisely consistent" with expected utility [see Pulley 1981]. The mainstream macroeconomist can rest assured that these firms are rational agents, or at least that only rational firms will survive, and can proceed with bigger questions about the market ecology of algorithms. However, the evidence from practitioners seems to show that maximization is not how these firms decide to allocate capital to HFT strategies, or make on/off decisions.

6. CAPABILITY SATISFICING

To understand how these firms actually make on/off decisions, I interviewed seven current or former employees of seven different algo/HFT[11] firms in Chicago in May through July of 2016. At the time of the interviews, one interviewee was a managing director at such a firm; two were Ph.D. quantitative strategists; one was a former quantitative strategist still working in the industry in a different capacity; one was a former director of quality assurance; and, two were traders. I cite these individuals anonymously as Interviewees A through G. In addition to the interviewees, I found quotes published in the specialist media. These quotes use real names plus the source reference. The conclusion, the aggregate dimension, is based on exploratory questions, which are appropriate for inter-disciplinary inquiry in management, technology, and finance [see Silverman, 2011].

I asked the HFT professionals: 1) Is Proposition 1 a reasonable representation of the decision-making problem in HFT? 2) How does your firm decide to turn on/off its trading strategies? 3) Do you backtest your strategies? 4) Is optimizing or satisficing a more apt description of how you make allocation decisions? 5) For how long do your strategies work? 6) Do you think about your collection of strategies as a portfolio? Using the data collected from these interviews, I follow Gioia et al. [2012] to documents first order concepts, from which I extract second order themes. These themes seem to support the aggregate dimension in Proposition 3.

Proposition 3: In behavioral finance theory and in practice, HFT firms are capability satisficers.

While the interviewees agreed that Proposition 1 is a reasonable problem definition, they suggested that they lack adequate information and resources to optimize their strategies or allocations. Thorough searches of all possible alternative strategies and all possible technological implementations are infeasible. Further, the moments of the reference distribution and demand cannot be known *ex ante*. As a result, they appear to form beliefs about the capability of their strategies heuristically, based upon their expertise, intuition, and competence. This is precisely the scenario described by Simon [see also Heath and Tversky, 1991; and Fox and Weber, 2002] and addressed in the AMH.

All of the interviewees were familiar with mean-variance optimization. What optimization does occur in HFT appears to be something different from utility maximization.

> *Interviewee C*: "[For cross-sectional HFT strategies[12]] you optimize to get the basket blends right and neutralize exposures to factors, but that is not utility maximization, which applies to signal processing, where you figure out which factors forecast best."

As a theme, HFT appears to be an ambiguous environment.[13] It is too complex and dynamic for optimization to occur.

Interviewee B: "You can't find a way to allocate using optimization [in HFT]."

Interviewee G: "You have to be constantly adapting. Otherwise, you just get lost."

Only one of the interviewees was aware of "satisficing" as a term, but the others offered as a concept that they search for strategies that are "good enough" or that cover their fixed costs.

Interviewee G: "In HFT, it's a race just to break even."

Once I explained satisficing, they agreed that it more accurately describes how they make allocation decisions.

Most of the interviewees agreed that the values of the variables presented in Table 1 cannot be known *ex ante*. In particular, the concept is that backtesting and simulated (or paper) trading are commonly through to provide little, if any, factual premises on which to base decisions, primarily due to overfitting and the dynamic nature of the environment.

> *Interviewee F*: "The world [both the market and the technology] changes so fast that backtesting has little relevance in HFT. Even if there is a marginal benefit, the cost and the time to build and run the backtesting engine are prohibitive."

Likewise, the long-term fixed costs and the returns to investments in research and technology are very difficult to forecast with any certainty. The market infrastructure evolves too quickly.

> *Interviewee E*: "If they build a new microwave tower [to transmit market data more quickly], then the whole trade may change."

Further, the concept arises that competitors' reactions to their strategies once implemented cannot be foreseen.

> *Interviewee D*: "HTF'ers are satisficers in a real sense, and not just because of problem complexity. …It's also because of game theory—the introduction of any strategy may be met with a counter-strategy…that defeats it."

Interviewees suggested that they form initial beliefs about the variables in Table 1 heuristically. From there, they appear to move on to (what I call) the concept of "probationary trading" (i.e. trading with limited scale or risk) in order to form the reference distribution *ex post*. This is similar to the "minimum viable product" approach used in innovation management [see

Moogk 2012], where a firm launches a basic version of a product or strategy into the market to gather "validated learning" used to justify continued development[14].

> *Mike Madigan*, CTO at WH Trading: "[We like to] stick a toe in the water and see if we can make money [DeFrancesco 2016]."

Once a strategy is up and running in some limited fashion, they appear to engage in probing and learning to improve capability, while scaling up the strategy. As a second order theme, probing and learning is a process used in ambiguous environments where firms "launch [a strategy] in the market, learn from failures, and modify for future attempts [Pich et al. 2002]." While the term "probing and learning" was unfamiliar to the interviewees, they agreed it accurately described how they work.

> *Interviewee F*: "You don't derive these strategies. They're based on logic about what other market participants will do. If we do this, then they'll do that, and we'll make money. You think, intuitively this [strategy] will work, let's turn it on, get some numbers and see… If you're not trading, you're not learning."
>
> *Interviewee D*: "Probing is precisely what these guys do."
>
> *Richard Gorelick*, co-founder of RGM Advisers LLC: "Our first day, we did four trades and made $17. Our second day, we lost a few hundred dollars and had to make adjustments."

Within a year, RGM's strategy was generating profits reliably enough to allow them to hire additional researchers [Foroohar 2010]. None of these comments suggests optimization.

> *Interviewee B*: "You don't stop doing something that isn't working too soon. You keep improving it until it works out. You don't give up [on a good idea] too early."

Interviewee A: “We turn it on and see what happens, then watch it, and add three or four more layers. …We get [the strategy] to a baseline acceptable level, then work on improving it. …You learn ten times more when something is running than during backtesting.”

The interviewees also suggested the prevalent use of selectionism, another second order theme. Selectionism is the process used in ambiguous environments where the firm searches through multiple alternatives to find the best one [see Pich et al. 2002; Sommer and Loch 2004]. The use of multiple trading teams within these firms fosters parallel and independent search in this regard [see for example Narang 2013; Lenglet 2011; and Davis et al. 2013].

Interviewee E: “We’re all dealing with a subset of the possible strategies…while we are constantly looking for the best ones, sometimes we settle for ones that are good enough.”

Interviewee B: “You don’t have unlimited risk capital. It’s like fishing. You pick where to put your hooks in the water. Hopefully, you catch some fish. If you’re [a bigger firm], you can put more hooks in the water.”

Interviewee G: “We used to run [multiple versions of] the same strategy with different configurations at the same time with one lots in a liquid market. Then, we would increase the size on the good ones.”

Relative to the methodology of Goia et al. [2005], Figure 6 organizes the concepts and themes drawn from these interviews to show how they support the aggregate dimension expressed in Proposition 3. Based upon the qualitative evidence, capability satisficing is an appropriate description of decision-making under uncertainty in HFT.

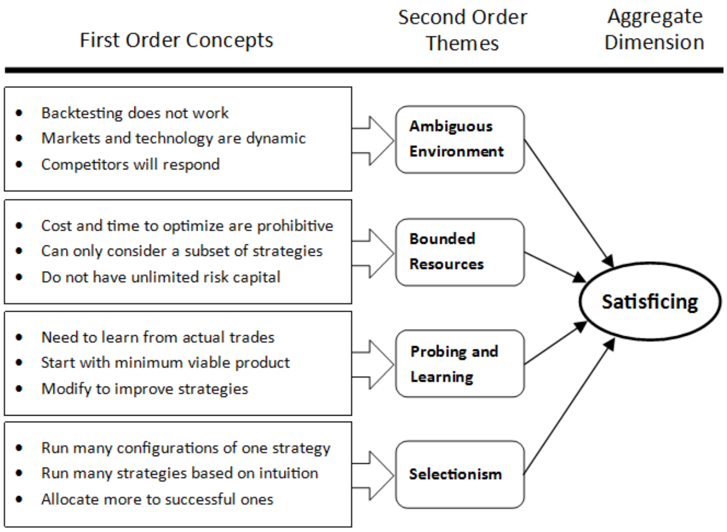

Figure 6: Structure of the Data

Relative to Brown and Sim [2009], because the SR^{HFT} is a satisficing measure and the C_{pl} is an affine transformation of the SR^{HFT}, then the C_{pl} in (3) is also satisficing measure. What Figure 6 implies is that if these firms are capability satisficers, then the aspiration level, or goal, of $C_{pl}(\ \xi, w\ ) = 1$ defines when a particular strategy is "good enough" to warrant an allocation. Thus, we can dismiss the common criticism of Simon's framework—that the satisficing point cannot be determined. However, in a complex and dynamic environment, what satisfices today may not satisfice tomorrow. Thus, these firms engage in a continuous struggle to maintain capability, or to "keep their head above water," as it were. This concept is also best described visually.

Turning again to the decomposition in (4), we can further tie the C_{pl} back to the intuition of the SR^{HFT} by noting that when $\varphi = 0$ and therefore $C_{pl}(\ \xi, w\ ) = 1$, the strategy is right at the aspiration level of one. In this case, the preference-dependent *critical* $SR^{HFT} = 1\ /\ CR$. Since the firm's tolerance for loss and time preference, and therefore its CR, *are* known *ex ante*, the firm searches for strategies with trading profit generating processes above the *critical* SR^{HFT}, the kind of water line, or are in a capability satisficing region (assuming fixed costs are known over the short-run). In the numerical example, where q = 1000, w = 20, z = 1.64, and c = 50, the *critical* $SR^{HFT} = 1 / (\sqrt{20} / 1.64) = 0.367$. Figure 7 shows the satisficing region for daily SR^{HFT}s over combinations of μ_w and σ_w.

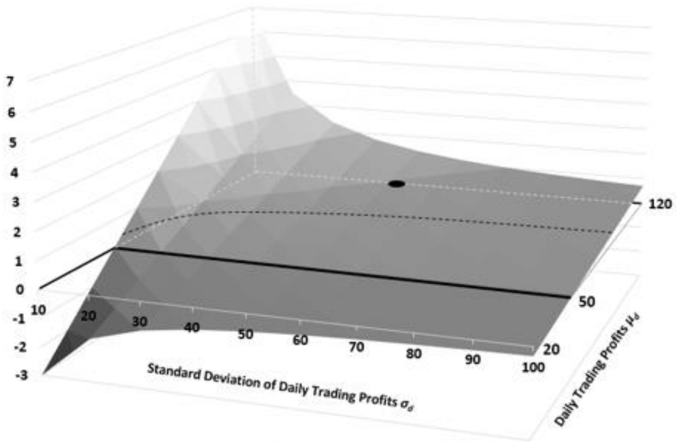

Figure 7: The Critical SR^{HFT} (1 / CR) Curve
q = 1000, ξ = .05, c = 50

In Figure 7, the heavy dark line is $SR^{HFT} = 0$, where $\mu_w = 50$ and $\pi_{net} = 0$. The black dotted line is the *critical* SR^{HFT} curve = 1 / CR = 0.367. Capability satisficing suggests that firms search for strategies with SR^{HFT}s above this curve. Once found, because the environment changes, the firm struggles to maintain the capability of a strategy through modification and reconfiguration based on heuristics, continually learning by receiving positive or negative feedback from the outcomes. From (3) we can also note that the level of investment in infrastructure (i.e. fixed costs c) over the long term (say annually) is such that $c_{252} \leq \mu_{252} - z\sigma_{252}$. Since μ and σ are not known *ex ante,* these firms' are making their up-front investments in the technological arms race based on competence and expertise, as in Kumiega and Van Vliet [2013].

However, "the heuristics of the old environment are not necessarily suited to the new [Lo 2007]," and this leads potentially to what might be perceived as maladaptive behavior, cognitive biases, or irrationality in their on/off allocation decisions. Thus, through the lens of biological approach and capability, "we can better understand the apparent contradictions between the EMH and the presence and persistence of behavioral biases [Lo 2007]."

7. CONCLUSION

This paper argues that capability is the appropriate problem representation of how HFT firms make decisions to allocate capital to strategies. While capability theory is consistent with expected utility theory, the qualitative evidence from practitioners suggests that 70% of the daily trading volume (for which HFT accounts) is driven by capability satisficers, not utility maximizers. Thus, there is a potential that behavioral biases may seep into algorithmic market agents through the heuristic on/off decisions that these firms make. This may result in a less than robust ecology and departures from rationality, which are thought to abound under the AMH.

This ought to have implications for how further studies into macroeconomic market efficiency approach the new ecology of algorithms. Under the AMH, the macroeconomist should be concerned with the supply of short-term information processing and liquidity as a function of the bounded rationality and *CR*s of the individual firms or their strategies. Likewise, levels of market efficiency and liquidity are dependent upon variable and fixed costs, which are passed along to longer-term traders and investors. The *CR* captures the firm's idiosyncratic time and risk preferences, and incorporates them into the C_{pl} allocation decision model. Consistent with the AMH, the firm may very well modify its *CR* over time in response to the changing environment. Thus, the C_{pl} defines the satisficing point through the inclusion of the *CR*, and enables moving back and forth between mainstream theory and the behavioral approaches based on industry practice. This methodology can be applied to any firm that makes algorithmic decisions under uncertainty.

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APPENDIX A: The Central Limit Theorem and HFT

Through the central limit theorem, the distribution of the sample mean *and* the distribution of the sample sum are both approximately normal when the sample size n is large. Any skewness and excess kurtosis disappear as n increases. For example, we usually annualize the *SR* by multiplying the expected daily log return by 252 and the standard deviation of the daily log returns by the $\sqrt{252}$. Thus, the annualized *SR* follows the distribution of the sample sum.

If we assume a random variable X from some infinite population with moments mean μ_X, variance σ_X^2, skewness γ_X, and the excess kurtosis κ_X. The sample sum Σ is simply the linear combination of the values X_i, for $i = 1,\ldots,n$. The moments of the distribution of Σ, the sample sum [see Finucan et al. 1974], and the moments of the distribution of $\bar{X}$, the sample mean [see Hinde 1986], are related to the moments of the distribution of X as shown in Table A1.

Moments of the Distribution of $\boldsymbol{X}$	Moments of the Distribution of the Sample Mean $\bar{\boldsymbol{X}}$	Moments of the Distribution of the Sample Sum $\boldsymbol{\Sigma}$
μ_X	$\mu_{\bar{X}} = \mu_X$	$\mu_\Sigma = \mu_X \cdot n$
σ_X^2	$\sigma_{\bar{X}}^2 = \sigma_X^2 / n$	$\sigma_\Sigma^2 = \sigma_X^2 \cdot n$
	$\sigma_{\bar{X}} = \sigma_X / \sqrt{n}$	$\sigma_\Sigma = \sigma_X \cdot \sqrt{n}$
γ_X	$\gamma_{\bar{X}} = \gamma_X / \sqrt{n}$	$\gamma_\Sigma = \gamma_X / \sqrt{n}$
κ_X	$\kappa_{\bar{X}} = \kappa_X / n$	$\kappa_\Sigma = \kappa_X / n$

Table A1: Relationships Among Distributions

From Table A1, we can note that although the standard deviation of the sample mean *decreases* as n increases where the standard deviation of the sample sum *increases* as n increases, the rates of change relative to the change in their respective means is the same, $\sqrt{n}$. What this means is that the value of any signal-to-noise ratio (such as the *SR* or C_{pl}) will be the same, regardless of whether we use the distribution of the sample mean or the sample sum of either profits or log returns. The equations in (A1) demonstrate the equivalence relationship, where the distribution of the sample *mean* $\bar{X}$ is on the left, and the distribution of the sample *sum* Σ is on the right.

$$\frac{\mu_{\bar{X}}}{\sigma_{\bar{X}}} = \frac{\mu_X}{\sigma_X} \cdot \frac{1}{\frac{1}{\sqrt{n}}} = \frac{\mu_X}{\sigma_X} \cdot \frac{\sqrt{n}}{1} \qquad \frac{\mu_{\Sigma}}{\sigma_{\Sigma}} = \frac{\mu_X}{\sigma_X} \cdot \frac{n}{\sqrt{n}} = \frac{\mu_X}{\sigma_X} \cdot \frac{\sqrt{n}}{1} \tag{A1}$$

APPENDIX B: Discussion of the Capability Ratio

If these firms are *ex ante* profit maximizers, then the probability of loss over w as $P(\pi_w < c_w)$ leads to the characterization of Proposition 1 as an optimization problem as in (B1), where the firm trades off π_{net} and ξ, discarding any optimal solution that fails the constraint.

$$\begin{aligned} &\text{max:} && \pi_{net} = \mu_w - c_w \\ &\text{subject to:} && P(\mu_w < c_w) \le \xi \end{aligned} \tag{B1}$$

Thus, we can back in to the C_{pl} quite simply as in (B2).

$$\begin{aligned} P(\mu_w < c_w) \le \xi \quad \equiv \quad & c_w \le Q_w(\xi) \\ & -c_w \ge -Q_w(\xi) \\ & \mu_w - c_w \ge \mu_w - Q_w(\xi) \\ GC_{pl}(\xi, w) = & \frac{\mu_w - c_w}{\mu_w - Q_w(\xi)} \ge 1 \end{aligned} \tag{B2}$$

The definition in (B2) is the generalized capability measure $GC_{pl}(\xi, w)$ of Cooper, et al. (2015) where Q is the inverse cumulative distribution function. Assuming normality of daily profits under the central limit theorem, this translates into the capability measure in (6). Thus, we could also view the firm as maximizing the reliability of net profits, 1 - $P(\pi_w < c_w)$.

If, on the other hand, these firms are satisficers, then the characterization of Proposition 1 as the capability ratio in (6) captures what these firms actually do in practice. That is, they first search for strategies that satisfy the constraint and then seek to increase π_{net} through probing and *ex post* validated learning.

[1] That is, assuming the strategy is running in control or to specification. See Kumiega and Van Vliet [2012].
[2] Expected utility theory conveniently assumes away these costs, largely because for longer-term investors they are relatively small.
[3] What counts as short-term is often not well defined. I will use it to mean something less than, say, 20 minutes. Strategies that hold positions less than this amount of time depend upon the earning the bid-ask spread (or at least not paying it) for profitability. Therefore, obtaining advantageous position in the limit order queue is a key source of competitive advantage, which necessitates the use of low latency (or high frequency) technology. See Cooper and Van Vliet [2015].
[4] In equity markets, exchanges often pay a fee, called a rebate, to trading firms that supply liquidity by placing limit orders in the limit order book.
[5] We may consider that the HFT strategy may run concurrently on or across multiple stocks or sets of stocks. This would entail an investigation not only of co-moments between the various distributions, but also of codependence among the inter-arrival processes. Such analysis is beyond the scope of this paper. Common practice in the industry in such cases is to look at the distribution of trading profits to the entire system (i.e. across all implementations) per unit of time, say per second [see Cooper and Van Vliet 2012]. This paper only considers a single implementation of an HFT strategy, though an explication of capability theory using units of time would unfold quite similarly.
[6] Given fixed costs of $1000 per month, we can back in to the fixed costs of 0.005 per share traded and 0.05 per trade, though these are not shown in Figure 1.
[7] Downside violations of the reference distribution should not occur due to real-time risk control [see for example Cooper and Van Vliet 2012].
[8] Or alternatively, whether the term structure of capability crosses one within an acceptable waiting time given an acceptable level of reliability $1 - \xi$. See Cooper, et al. [2015].
[9] Firms may be able to improve on the normality assumption and use other distributions. Nevertheless, the same methodology idea of percentile criteria for decision-making would apply using the methodology of Cooper et al. [2015].
[10] Measuring these variables is beyond the scope of this paper.
[11] Many individuals declined to be interviewed. Those who did agree would not divulge whether their strategies counted as high frequency or were merely algorithmic, so I refer to their firms as algo/HFT. In addition, some of the interviewees preferred to talk in the third person, rather than give the appearance of divulging firm-specific information.
[12] Such as index arbitrage.
[13] That is, unmeasurable uncertainties as in Knight [1921].
[14] That is, viability does not imply capability.